DISCRETE ADVERSARIAL ATTACKS AND SUBMODULAR OPTIMIZATION WITH APPLICATIONS TO TEXT CLASSIFICATION

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ABSTRACT
Adversarial examples are carefully constructed modifications to an input that completely change the output of a classifier but are imperceptible to humans. Despite these successful attacks for continuous data (such as image and audio samples), generating adversarial examples for discrete structures such as text has proven significantly more challenging. In this paper we formulate the attacks with discrete input on a set function as an optimization task. We prove that this set function is submodular for some popular neural network text classifiers under simplifying assumption. This finding guarantees a $1 - 1/e$ approximation factor for attacks that use the greedy algorithm. Meanwhile, we show how to use the gradient of the attacked classifier to guide the greedy search. Empirical studies with our proposed optimization scheme show significantly improved attack ability and efficiency, on three different text classification tasks over various baselines. We also use a joint sentence and word paraphrasing technique to maintain the original semantics and syntax of the text. This is validated by a human subject evaluation in subjective metrics on the quality and semantic coherence of our generated adversarial text.

1 INTRODUCTION
Adversarial examples are carefully constructed modifications to an input that completely change the output of a classifier but are imperceptible to humans. Spam filtering and the carefully-crafted emails designed to fool these early classifiers are the first examples of adversarial machine learning going back to 2004 (Dalvi et al., 2004; Lowd & Meek, 2005); see also the comprehensive survey by Biggio et al. (Biggio & Roli, 2017). Szegedy et al. (Szegedy et al., 2013) discovered that deep neural network image classifiers can be fooled with tiny pixel perturbations; exploration of this failure of robustness has received significant attention recently, see e.g. (Goodfellow et al., 2014; Moosavi Dezfooli et al., 2016; Papernot et al., 2016b; Carlini & Wagner, 2017; Evtimov et al., 2017; Chen et al., 2017; Su et al., 2018). Adversarial training (Goodfellow et al., 2014; Madry et al., 2017) seems to be the state of the art in defense against adversarial attacks, but creating robust classifiers remains challenging, especially for large image classifiers, see e.g. Athalye at al. (Athalye et al., 2018).

Despite these successful attacks for continuous data (such as image and audio samples), generating adversarial examples for discrete structures such as text and code has proven significantly more challenging in two aspects:

One challenge is how to develop a fast yet (provably) effective attacking scheme. Gradient-based adversarial attacks for continuous data no longer directly apply to discrete structures. Although some variants are proposed when the model is differentiable to the embedding layer (Papernot et al., 2016a; Li et al., 2016; Ebrahimi et al., 2017; Gong et al., 2018), this line of methods achieve efficiency but suffer from poor success rate.

 Meanwhile, another natural idea is to find feasible replacement for individual features like words or characters. However, since the space of possible combinations of substitutions grows exponentially with the length of input data, finding the optimal combination of substitutions is intractable. Recent heuristic attacks on NLP classifiers operate by greedy character-level or word-level replacements (Ebrahimi et al., 2017; Kuleshov et al., 2018; Yang et al., 2018). However, greedy methods are usually slow, and it’s theoretically not understood when they achieve good performance.

The other issue is how to maintain the original functionality of the input. Specifically for text, it remains challenging to preserve semantic and syntactic properties of the original input from the point of view of a human. Existing methods either require to change too many features, or change the original meaning. For instance, (Kuleshov et al., 2018) alters up to 50% of words in each input document to achieve a
With the proposed optimization scheme, we show significant improved attack performance over most recent baselines. Meanwhile we propose a joint sentence and word paraphrasing technique to simultaneously ensure retention of the semantics and syntax of the text.

2 RELATED WORK

Broadly speaking, adversarial examples refer to minimally modified natural examples that are spurious but perceptually similar and that lead to inconsistent decision making between humans and machine learning models. An example is automatically classifying an adversarial stop sign image (according to humans) as a speed limit sign. For continuous data such as images or audio, generating adversarial examples is often accomplished by crafting additive perturbations of natural examples, resulting in visually imperceptible or inaudible noise that misleads a target machine learning model. These small yet effective perturbations are difficult for humans to detect, but will cause an apparently well-trained machine learning model to misbehave; in particular, neural networks have been shown to be susceptible to such attacks (Szegedy et al., 2013), giving rise to substantial concern about safety-critical and security-centric machine learning applications.

For classifiers with discrete input structures, a simple approach for generating adversarial examples is to replace each feature with similar alternatives. Such features for text classification tasks are usually individual words or characters. Such attacks can be achieved using continuous word embeddings or with respect to some designed score function; this approach has been applied to attack NLP classifiers (Papernot et al., 2016a; Li et al., 2016; Miyato et al., 2016; Samanta & Mehta, 2017; Liang et al., 2017; Yao et al., 2017; Gong et al., 2018; Kuleshov et al., 2018; Gao et al., 2018; Alzantot et al., 2018; Yang et al., 2018) and sequence-to-sequence models (Ebrahimi et al., 2017; Wong, 2017; Zhao et al., 2018; Cheng et al., 2018). The work in (Ribeiro et al., 2018) considers semantically equivalent rules for debugging NLP models, but under the same input structure. This is a natural but limited practice to only consider attacks within one input structure, namely word or characters, but no joint attacks, nor the effect incurred from sentences. Unlike prior work, we conduct a joint sentence and word paraphrasing technique. It considers sentence-level factors and allows more degrees of freedom in generating text adversarial examples, by exploring the rich set of semantically similar paraphrased sentences.

Jia and Liang studied adversarial examples in reading comprehension systems by inserting additional sentences (Jia & Liang, 2017), which is beyond the concept of this paper since the approach changes the original meanings. Another related line of research, although not cast as adversarial examples, focuses on improving model robustness against out-of-vocabulary terms (Belinkov & Bisk, 2017) or obscured embedding space representations (Mrkšić et al., 2016).

3 PRELIMINARY

In this paper, we propose a general framework for generating adversarial examples with discrete input data. A collection of such data and corresponding attacks are presented in Table 1.

To present our mathematical formulation, we start by introducing some notation.

Input Structure. Let the input $x = [x_1, x_2, \cdots, x_n] \in \mathbb{R}^n$.
I suppose I should write a review here since my little Noodle-oo is currently serving as their spokes dog in the photos. We both love Scooby Do’s. They treat my little butt-faced dog like a prince and are receptive to correcting anything about the cut that I perceive as being weird. Like that funny poopy pompadour. Mohawk it out, yo. Done. In like five seconds my little man was looking fabulous and bad ass. Not something easily accomplished with a prancing pup that literally chases butterflies through tall grasses. (He ended up looking like a little lamb as the cut grew out too. So adorable.) The shampoo they use here is also amazing. Noodles usually smells like tacos (a combination of beef stalk and corn chips) but after getting back from the Do’s, he smelled like Christmas morning! Sugar and spice and everything nice instead of frogs and snails and puppy dog tails. He’s got some gender identity issues to deal with. The pricing is also cheaper than some of the big names below. I’m talking to you Petsmart! I’ve taken my other pup to Smelly Dog before, but unless I need dog sitting play time after the cut, I’ll go with Scooby’s. They genuinely seem to like my little Noodle monster.

Task: Spam Filtering. Classifier: WCNN. Original label: 100% None-spam. ADV label: 100% Spam

Task: Fake-News Detection. Classifier: LSTM. Original label: 100% Fake. ADV label: 77% Real

Figure 1. Examples of generated adversarial examples. The color red denotes sentence-level paraphrasing, and blue denotes word-level paraphrasing.

<table>
<thead>
<tr>
<th>input data</th>
<th>task</th>
</tr>
</thead>
<tbody>
<tr>
<td>document</td>
<td>text classification</td>
</tr>
<tr>
<td>code</td>
<td>malware detection</td>
</tr>
<tr>
<td>url address</td>
<td>malicious website check</td>
</tr>
</tbody>
</table>

Table 1. Applications to the framework.

$\mathcal{X}^n$ be a list of $n$ features (might be padded). For text environment, the feature space $\mathcal{X}$ can be the character, word, phrase, or sentence space. For the problem of malware detection, $x$ is a concatenation of code pieces.

**Remark 1.** For concrete usage, we use $w \in \mathcal{W}$ to denote word space, and $s \in \mathcal{S}$ to denote sentences to distinguish the differences.

**Embedding $V$.** The embedding layer is a key transition from discrete input data into continuous space, which could then be fed into the classifier. For text domain, we typically use the bag-of-words embedding or word-to-vector embedding.

For a bag-of-words embedding, $V : \mathcal{X}^n \rightarrow \mathbb{R}^D$ represents a document as the statistics of word counts, i.e., the summation of each word’s one-hot representation. Meanwhile, word-to-vector embeddings characterize different words as $D$-dimensional vectors, i.e., $V(x) \in \mathbb{R}^D, \forall x \in \mathcal{X}$. When there’s no ambiguity, we also use $V : \mathcal{X}^n \rightarrow \mathbb{R}^{n \times D}$ to denote the concatenation of word vectors of the input document as a list of words.

**Transformation Indexing.** Suppose each feature $x \in \mathcal{X}$ has (at most) $k - 1$ possible replacements, denoted by $x^{(i)}$, $i \in [k - 1] \equiv \{1, 2, \ldots, k - 1\}$. For future use, we also define $x^{(0)} = x, \forall x \in \mathcal{X}$.

A valid transformation $T$ is the combined replacement of each individual feature $x_i$, $i \in [n]$. Therefore we index $T$ by a vector $l \in \{0, 1, \ldots, k - 1\}^n$, and $l_i$ indicates the index of each replacement $i$. Namely, $T_i(x = [x_1, x_2, \ldots, x_n]) = [x_1^{(l_1)}, x_2^{(l_2)}, \ldots, x_n^{(l_n)}]$. An example with word replace-
ment in the text classification environment can be found in Figure 2.

![Figure 2. An illustration of the transformation indexing when applying to a text sentence. In this example, the transformation denoted as 1 modifies the original sentence to the new one shown in the red boxes.](Image)

**Classiﬁer output** $C_y$. We consider a targeted attack, i.e., we want to maximize the output probability $C$ over a speciﬁc target label $y$.

In this paper, we use a regular lower-case symbol to denote a scalar or a single feature, and use a bold lower-case symbol for a vector or a list of features.

### 3.1 Problem Setup

In most scenarios, we only allow transformations on at most $m$ features, then the constraint is $\|l\|_0 \leq m$. Therefore we present the adversarial attack problem formally:

**Problem 1.** For some input data $x \in X^n$ and target label $y$, we try to find a feasible transformation $T_l$, where $l^* \in \{0, 1, \cdots, k - 1\}^n$ is the index so that:

$$l^* = \arg \max_{\|l\|_0 \leq m} C_y(V(T_l(x))).$$

(1)

Or similarly, we want to find the set of features to attack, i.e.,

$$S^* = \arg \max_{|S| \leq m} f(S),$$

(2)

where we defined the set function $f : 2^{[n]} \to \mathbb{R}$, $f(S) = \max_{s \subseteq S} C_y(V(T_s(x)))$.

The set function $f(S)$ represents the classiﬁer output for the target label $y$ if we apply a set of transformations $S$. We are therefore searching over all possible sets of up to $m$ replacements to maximize the probability of the target label output of a classiﬁer.

**Remark 2.** **In this paper, we focus on replacements via word and sentence paraphrasing for empirical studies. However, our formulation is general enough to represent any set of discrete transformations. Possible transformations include replacement with the nearest neighbor of the gradient direction (Gong et al., 2018) and word vectors (Kuleshov et al., 2018), or ﬂipping characters within each word (Ebrahimi et al., 2017). We will also conduct a thorough experimental comparisons among different choices.**

### 4 Theoretical Analysis

First, notice that the original problem is computationally intractable in general:

**Proposition 1.** For a general classiﬁer $C_y$, the problem 1 is NP-hard. Speciﬁcally, even for some convex $C_y$, the problem 1 can be polynomially reduced to subset sum and hence is NP-hard.

Details and all proofs referenced to in this paper can be found in the Appendix.

#### 4.1 Unifying Related Methodology via Further Assumptions

Fortunately, with further assumptions it becomes possible to solve problem 1, above, in polynomial time. Some existing heuristics are proposed to generate adversarial examples for the text classiﬁcation problem. Though usually not speciﬁcally proposed in the relevant literature, we unify the underlying assumptions for these heuristics to succeed in polynomial time in this section.

One possible assumption is that the original function $C_y$ is smooth, which could afterwards be approximated by its first-order Taylor expansion:

$$C_y(V(T_l(x))) = C_y(v) + \langle \nabla C_y(v), V(T_l(x)) - v \rangle + O(\|V(T_l(x)) - v\|_2^2)$$

where $v = V(x)$. Therefore, Problem 1 can be relaxed as follows:

**Problem 2.** Given gradient $\nabla C_y(v)$, where $v = V(x)$, maximize function $C_y$ by its ﬁrst-order Taylor expansion:

$$l^* = \arg \max_{\|l\|_0 \leq m} V(T_l(x))^\top \nabla C_y(v).$$

(3)

Problem 2 is similar to the Frank-Wolfe method (Frank & Wolfe, 1956) in continuous optimization and is easy to solve:

**Proposition 2.** Problem 2 can be solved in polynomial time for both bag-of-words and word to vector embeddings. Speciﬁcally, $f(S) = \arg \max_{s \subseteq S} V(T_s(x))^\top \Delta C_y(v)$ can be written as $\sum_{s \subseteq S} w_s$ for some $w$ irrelevant to $S$, where $v = V(x)$.

Related methods like (Gong et al., 2018) are attempts to solve problem 2. They propose to conduct transformations via replacement by synonyms chosen by (3). However, activations like ReLU break the smoothness of the function, and ﬁrst order Taylor expansion only cares about very local information, while embeddings for word synonyms could be actually not that close to each other. Consequently, this unnatural assumption prevents related gradient-based attacks to achieve good performance.
Besides smoothness, another more natural assumption is that \( f(S) \) in the original problem 1 is submodular (Narayanan, 1997; Fujishige, 2005). Submodular is a property that is defined for set functions, which characterizes the diminishing returns of the function value change as the size of the input set increases.

**Definition 1.** (Schrijver, 2003) If \( \Omega \) is a finite set, a submodular function is a set function \( f: 2^\Omega \to \mathbb{R} \), where \( 2^\Omega \) denotes the power set of \( \Omega \), which satisfies one of the following equivalent conditions.

1. For every \( X, Y \subseteq \Omega \) with \( X \subseteq Y \) and every \( x \in \Omega \setminus Y \) we have that \( f(X \cup \{x\}) - f(X) \geq f(Y \cup \{x\}) - f(Y) \).
2. For every \( S, T \subseteq \Omega \) we have that \( f(S) + f(T) \geq f(S \cup T) + f(S \cap T) \).
3. For every \( X \subseteq \Omega \) and \( x_1, x_2 \in \Omega \setminus X \) we have that \( f(X \cup \{x_1\}) + f(X \cup \{x_2\}) \geq f(X \cup \{x_1, x_2\}) \).

With the design of \( f(S) \) in Problem 1 to be monotone non-decreasing and if we further assume \( f \) to be submodular, our task becomes to maximize a monotone submodular function subject to a cardinality constraint (Nemhauser et al., 1978). Therefore, greedy method guarantees a good approximation of the optimal value of Problem 1:

**Claim 1.** In problem 1, \( f \) is monotone non-decreasing. Furthermore, if the function \( f \) is submodular, greedy methods achieve a \((1 - 1/e)\)-approximation of the optimal solution in polynomial time.

Both our work and the optimization scheme from (Kuleshov et al., 2018) propose some variants of greedy methods with the underlying submodular assumption.

The greedy method proposed in (Kuleshov et al., 2018) selects candidate replacements directly by function value, one word at a time, which we will refer as the objective-guided greedy method. We will propose a more efficient yet comparable effective greedy method that is guided by the gradient magnitude in Section 5.2, and compare with the above two methods in Section 6.3. As an extension from the continuous optimization, our method uses the well-studied Gauss-Southwell rule (Nutini et al., 2015) that is provably better than random selection. In each iteration, we determine and select the most important words by the gradient norm of words’ embeddings, and then find the greediest transformation within the search space of the selected words. The advantage is that we are able to conduct multiple replacements in one iteration and thus take into consideration the joint effect of multiple words replacements. We will introduce our method, which we call Gradient-Guided Greedy Word Paraphrasing in Algorithm 3, and will show empirical performance comparison with the (objective-guided) greedy method (Kuleshov et al., 2018) and the gradient method used in (Gong et al., 2018) in Section 6.3.

**4.2 Submodular Neural Networks on the Set of Attacks**

To argue that submodular is a natural assumption, we study and summarize the neural networks are submodular on the set of attacks.

In (Bilmes & Bai, 2017), it provides a class of submodular functions used in the deep learning community called deep submodular functions. Nevertheless the deep submodular functions are not necessarily applicable to our set function. We hereby formally prove the following two kinds of neural networks, that are ubiquitously used for text classification, indeed satisfy submodular property on the set of attacks under some conditions.

**Simplified W-CNN (Kim, 2014)**

Denote the stride as \( s \), the number of grams (window size) \( h \), and the word vector of the \( i \)-th word in a document as \( v_i \equiv V(x_i) \). Then the output for the convolutional layer is a matrix \( C = [c_{1j}]_{j \in [n/s], i \in [m]} \) from \( n \) words and \( m \) filters:

\[
c_{ij} = \phi(w_j^T v_{s(i-1)+1:s(i-1)+h} + b_j), \quad i = 1, 2, \ldots, n/s,
\]

where \( w_j \in \mathbb{R}^{Dh} \) is the \( j \)-th filter, \( b_j \) is the corresponding bias term and \( \phi \) is the non-linear, and non-decreasing activation such as ReLU, tanh and sigmoid function. \( v_{ij} \) denotes the concatenation of word vectors in the window of words \( i \) through \( j \), namely \( [v_i^T, v_{i+1}^T, \ldots, v_j^T]^T \in \mathbb{R}^{D(j-i+1)} \). Each filter \( w_j \) is applied to individual windows of words to produce a feature map \( c' = [c_{11}, c_{21}, \ldots, c_{n/s, 1}]^T \).

Afterwards, a max-over-time pooling is applied to each feature map to form the penultimate layer \( \tilde{c} = [\tilde{c}_1, \tilde{c}_2, \ldots, \tilde{c}_m] \), where \( \tilde{c}_j \) is the largest value in \( c' \):

\[
\tilde{c}_j = \max_i c_{ij}.
\]

Compared to the original (Kim, 2014) paper, we only omit
the dropout and softmax layer, and instead consider the following WCNN classifier output for a target label:

\[ C_{\text{WCNN}}(v_1:n) = w' \cdot \hat{c} + b' \] (4)

**Theorem 1.** We consider the simple version of WCNN classifier described in (4), and suppose there’s no overlapping between each window, i.e., \( s \leq h \), and \( w' \) has all non-negative values. If further we only look at transformations that will increase the output, i.e., \( w'_j V(x_t^{(i)}) \geq w'_j V(x_t), \forall i \in [n], j \in [m], t \in [k-1] \), then \( f_{\text{WCNN}}(S) = \max_{\text{supp}(S) \subseteq S} C_{\text{WCNN}}(V(T_i(x))) \) is submodular.

The proof sketch is as follows. Every coordinate in \( \hat{c} \) is a combination of max pooling over a modular function and is therefore submodular. And finally sums of submodular functions is still submodular.

Besides word-level CNN, another network that is popular in the NLP community is the recurrent neural network (RNN) or its variants. We will show that under some conditions, RNN satisfies submodular property.

**Recurrent Neural Network with One-dimensional Hidden Units**

Consider a RNN with \( T \) time steps and each hidden layer is a single node. Then for all \( t \leq T \), given the value of a previous hidden state \( h_{t-1} \in \mathbb{R} \) and an input word vector \( v_{t-1} \in \mathbb{R}^L \) (\( v_{t} \equiv V(x_t) \)), RNN computes the next hidden state \( h_t \) and output vector \( o_t \in \mathbb{R} \) as:

\[ h_t = \phi(w h_{t-1} + m^T v_{t-1} + b) \] (5)

The classifier output is \( C_{\text{RNN}}(v_{1:T}) = y h_T \).

**Theorem 2.** For a recurrent neural network with \( T \) time steps and one-dimensional hidden nodes described in (5), if \( w \) and \( y \) are positive, and the activation is a non-decreasing concave function, then \( f_{\text{RNN}}(S) = \max_{\text{supp}(1) \subseteq S} C_{\text{RNN}}(V(T_i(x))) \) is submodular.

This result is quite surprising, since the word vectors influence the network’s output on different time steps and are by no means separable. In the proof, we first show that a same amount of change induced on an intermediate layer has a diminishing effect when the network is attacked on more features. Then together with the concavity and non-decreasing property of the network, we are able to finish the proof.

5 **Adversarial Text Examples via Paraphrasing**

In order to conduct adversarial attacks on models with discrete input data like text, one essential challenge is how to select suitable candidate replacements so that the generated text is both semantic meaning preserving and syntactically valid. Another key issue is how to develop an efficient yet effective optimization scheme to find good transformations. To solve the above two issues, we now propose our methodology for generating adversarial examples for text.

5.1 **Joint Sentence and Word Paraphrasing**

To coincide with our definition of adversarial examples for text, we need to determine appropriate word and sentence paraphrasing methods that maintain the semantic meaning of the original text. Our scheme is to generate an initial set for word and sentence replacements with a well-studied paraphrasing corpus and then filter out discrepant choices based on their semantic and syntactic similarities to the original text. Similar mechanism was also used by (Kuleshov et al., 2018) to generate word replacement candidates.

**Paraphrasing Corpus.** For word paraphrasing, we use the Paragram-SL999 (Wieting et al., 2015) of 300 dimensional paragram embeddings to generate neighboring paraphrasing for words. For sentences, we use the pretrained model from Wieting and Gimpel’s Para-nmt-50m project (Wieting & Gimpel, 2017) to generate sentence paraphrases.

We further specify semantic and syntactic constraints to ensure good quality in adversarial texts:

**Semantic similarity.**

We use the Word Mover Distance (WMD) (Kusner et al., 2015; Wu et al., 2018) to measure semantic dissimilarity. For sentence pairs, WMD captures the minimum total semantic distance that the embedded words of one sentence need to “travel” to the embedded words of another sentence. While for words, WMD directly measures the distance between their embeddings.

**Syntactic similarity.**

Alongside the semantic constraint, one should also ensure that the generated sentence is fluent and natural. We make use of a language model as in (Kuleshov et al., 2018), \( P: \mathcal{X}^n \rightarrow [0, 1] \) to calculate the probability of the adversarial sentence, and require:

\[ |\ln(P(x)) - \ln(P(x'))| \leq \delta, \]

where \( x' \) is the adversarial sentence paraphrased from \( x \).

In Algorithm 1, we present the whole procedure of finding the neighboring sets to conduct our proposal joint sentence and word paraphrasing attack. While with more details, we show how to use the objective value as well as gradient information to guide the search in Algorithm 2 (for sentences) and 3 (for words).

5.2 **Gradient-Guided Greedy Method**

In Section 3.1 we have demonstrated the difficulty of finding the best transformation from combinatorially many choices.
Algorithm 1 Joint Sentence And Word Paraphrasing \((C_y, x^{(0)}, \pi, \delta, \lambda_s, \lambda_w, \delta_s, \delta_w, \tau, k)\)

1: **Input:** Classifier \(C_y\) associated with target label \(y\), input document \(x^{(0)}\), language model \(\pi\) trained on the training set, syntactic threshold \(\delta_s\), sentence and word paraphrasing ratio \(\lambda_s, \lambda_w\), termination threshold \(\tau\), WMD threshold \(\delta_s, \delta_w\), limit number of paraphrases \(k\).
2: Conduct sentence separation \(x^{(0)} \rightarrow [s_1, s_2, \cdots, s_l], s_i \in S, 1 \leq i \leq l\). (See Remark 1).
3: Create sentence neighboring set \(S = \{S_1, S_2, \cdots, S_l\}\), where each \(S_i \subset S\) satisfies that \(|S_i| \leq k\) and \(WMD(s_i, s) \leq \delta_s, \forall s \in S_i\).
4: \(x^{(1)} \leftarrow \text{Greedy Sentence Paraphrasing}(C_y, x^{(0)}, S, \lambda_s, \tau)\) in Alg. 2.
5: If \(C_y(V(x^{(1)})) \geq \tau\) Return \(x^{(1)}\).
6: Conduct word separation \(x^{(1)} \rightarrow [w_1, w_2, \cdots, w_n], w_i \in W, 1 \leq i \leq n\).
7: Create word neighboring set \(W = \{W_1, W_2, \cdots, W_n\}\), where each \(W_i \subset W\) satisfies that \(|W_i| \leq k\) and \(WMD(w_i, w) \leq \delta_w, |P(x^{(1)}(w)) - P(x'((w)))| \leq \delta, \forall w \in W_i\), where \(x'((w))\) is text \(x^{(1)}\) in which \(w_i\) is substituted by \(w\).
8: \(x^{(2)} \leftarrow \text{Gradient Guided Greedy Word Paraphrasing}(C_y, x^{(1)}, W, \lambda_w, \tau)\) in Alg. 3.
9: Return \(x^{(2)}\).

Algorithm 2 Greedy Sentence Paraphrasing \((C_y, x, S, \lambda_s, \tau)\)

1: **Input:** Document \(x\) as list of sentences \([s_1, s_2, \cdots, s_l]\), sentence neighboring sets \(S = \{S_1, S_2, \cdots, S_n\}\), model \(C_y\) and parameters \(\lambda_s, \tau\).
2: While \(C_y(V(x)) \leq \tau\) and number of sentence paraphrased \(\leq \lambda_s l\) do
3: Create candidate set \(M = \emptyset\).
4: For \(j = 1, 2, \cdots, l\) do
5: For \(s \in S_j\) do
6: Substitute \(s_i\) by \(s\) to get \(x'\) and add it to the candidate set \(M \leftarrow M \cup \{x'\}\).
7: End for
8: \(x \leftarrow \arg \max_{x' \in M} C_y(V(x'))\).
9: End for
10: End while

Here we specify our proposal, gradient-guided greedy word paraphrasing, as shown in Algorithm 3. We can see that we first use gradient values to determine the index set of \(N\) words \(([w_{i_1}, w_{i_2}, \cdots, w_{i_N}]\) that we want to replace (steps 4-5). Then in steps 7-15 we create a candidate set of all possible transformations in \([W_1, W_2, \cdots, W_N]\). Finally, we choose the best paraphrase combinations within the candidate set. In this way, we are able to conduct multiple replacements in one iteration. Iteration and thus take into consideration the joint effect of multiple words replacements.

This method is based on an intuition derived from coordinate descent with the Gauss-Southwell rule (Nutini et al., 2015) in the continuous optimization theory; normally, updating the coordinates with the highest absolute gradient values is provably faster than optimizing over random coordinates (Lei et al., 2016; 2017). We only conduct this method in word paraphrasing, since the gradient information of sentence embedding is less trustworthy. Usually sentence paraphrasing changes the number of words. The calculated gradient before paraphrasing step might not even correspond to the right position of the new sentence. Therefore it makes more sense to use the objective value only and goes back to our Algorithm 2.

6 EXPERIMENTS

In this section, we provide empirical evidence of the advantages of our attack scheme via joint sentence and word paraphrasing on both two WCNN and LSTM models and various classification tasks. Our code for replicating our experiments is available online\(^1\).

6.1 Tasks and Models.

We focus on attacking the following state-of-the-art models which also echo our theoretical analysis:

**Word-level Convolutional Network (WCNN).**

We implement a convolutional neural network (Kim, 2014) with a temporal convolutional layer of kernel size 3 and a max-pooling layer, followed by a fully connected layer for the classification output.

**Long Short Term Memory classifier (LSTM).**

The LSTM Classifier (Hochreiter & Schmidhuber, 1997) is well-suited to classifying text sequences of various lengths. We construct a one-layer LSTM with 512 hidden nodes, following the architecture used in (Kuleshov et al., 2018; Zhang et al., 2015).

We carried out experiments on three different text classification tasks: fake-news detection, spam filtering and sentiment analysis; these tasks are also considered in (Kuleshov et al., 2018). The corresponding datasets include:

**Fake/Real News.**

\(^1\)https://github.com/cecilialeiqi/adversarial_text
The fake news repository (McIntire, 2017) contains 6336 clean articles of both fake and real news in a 1:2 ratio (5336 training and 1000 testing), with both left- and right-wing sites as sources.

**Trec07p (emails).**
The TREC 2007 Public Spam Corpus (Trec07p) contains 75,419 messages of ham (non-spam) and spam in a 1:2 ratio. We preprocess the data and retain only the main content in each email. We randomly hold out 10% as testing data.

**Yelp reviews.**
The Yelp reviews dataset was obtained from the Yelp Dataset Challenge in 2015. The polarity dataset we used was constructed for a binary classification task that labels 1 star as negative and 5 star as positive. The dataset contains 560,000 training and 38,000 testing documents.

### 6.2 General Settings

For the training procedure, we use similar settings for the WCNN and LSTM classifier. We extracted the top 100,000 most frequent words to form the vocabulary. The first layer of both WCNN and LSTM is the embedding that transforms individual word into a 300-dimensional vector using the pretrained word2vec embeddings (Mikolov et al., 2013). We randomly hold out 10% training data as validation set to choose the number of epochs and use a constant mini-batch size of 16.

We manually selected the hyperparameters for each dataset. We set the termination threshold $\tau = 0.7$, and set a neighbor size $k$ for possible paraphrases to be 15. We set the semantic similarity $\delta = 0.75$ for all datasets and syntactic bound $\delta_2 = 2$ for news and yelp datasets, and $\delta = \infty$ for Trec07p; the email dataset contains many corrupted words rendering the language model ineffective. For all datasets, we only allow $\lambda_w = 20\%$ word paraphrasing. We set the sentence paraphrasing ratio $\lambda_s = 20\%$ for yelp and news dataset, and for spam $\lambda_s = 60\%$.

### 6.3 Accuracy comparisons.

After setting up the experimental environment, we now present the empirical studies in several aspects. In Table 2 we present the original and adversarial test accuracy on the

<table>
<thead>
<tr>
<th>Dataset</th>
<th>WCNN</th>
<th>LSTM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Origin</td>
<td>ADV (ours)</td>
</tr>
<tr>
<td>News</td>
<td>93.1%</td>
<td>35.4%</td>
</tr>
<tr>
<td>Trec07p</td>
<td>99.1%</td>
<td>48.6%</td>
</tr>
<tr>
<td>Yelp</td>
<td>93.6%</td>
<td>23.1%</td>
</tr>
</tbody>
</table>

Table 2. Classifier accuracy on each dataset. Origin and ADV respectively stand for the clean and adversarial testing results. For all datasets, we set word paraphrasing ratio to be $\lambda_w = 20\%$ for our method (ADV(ours)). We include results from (Kuleshov et al., 2018) for comparison. The first column indicates reported values in their paper; while the consequent column marked by asterisk is our implementation using greedy method in (Kuleshov et al., 2018) and the same word neighboring set as our method. Both results use large $\lambda_w = 50\%$ and allow many more word replacements.

The similarity is in $[0,1]$ basis where 1 means identical and 0 means complete irrelevant.

---

1. We use the WMD similarity in python’s spacy package.
Table 3. Attack success rate (denoted by SR) and time comparisons of each optimization mechanism. The performance is reported on the WCNN classifier. Here objective-guided greedy indicates the greedy method used in (Kuleshov et al., 2018), and the gradient method is the one suggested in (Gong et al., 2018). We can see that even when only applying Algorithm 3, our optimization method is more effective among others.

three datasets with the two chosen models, where we allow 20% word replacements. We also include the presented adversarial accuracy from (Kuleshov et al., 2018) for reference. Since the word neighboring sets for the two methods are different and the values are not directly comparable, one might argue that we have broaden the search space of words to make the problem easier. Therefore we also implemented the greedy mechanism in (Kuleshov et al., 2018) using the same word replacement set as our method has chosen (marked by *). Both the reported values from (Kuleshov et al., 2018) and our implementation allow 50% word replacements. From Table 2 we can see that in both settings, we are able to successfully flip more prediction classes with fewer word paraphrases. We hereby conclude that joint sentence and word level attacks is much more effective than mere word replacements. Meanwhile, since sentence-level attacks almost perfectly preserve the original meaning, our method can be less susceptible to humans. In the Appendix we use some concrete examples to show the significantly improved quality of our generated adversarial texts compared to (Kuleshov et al., 2018; Gong et al., 2018). In the examples, we can see that sometimes by simplifying or changing the language, or even by making the slightest changes like adding or erasing space, the sentence paraphrase can make a tremendous difference to the classifier output. Consequently, our method does far fewer word level alterations than other methods and greatly reduces the possibility of syntactic or grammar errors.3

To further investigate the joint effect from combining sentence and word level attacks, we also study how each model is susceptible to different degrees of change permitted for both attack levels. Therefore we tested and presented the joint influence in Figure 4 for ratios of sentence paraphrasing $\lambda_s$ ranging from 0% to 60%, as well as for allowed word paraphrasing percentages $\lambda_w$: 0%, 10%, 20% and 30%. In all datasets, sentence paraphrasing is especially effective when we allow only a few word paraphrases. For instance, in the sentiment analysis task, we could only successfully attack $\sim 5\%$ reviews by paraphrasing 10% of words. But after conducting 60% sentence paraphrasing beforehand, the success rate increases to almost 60%.

6.4 Optimization Method Comparisons for Word-level Attacks.

To investigate the effectiveness of our proposed gradient-guided greedy method, we implement and compare the time consumption and success rate with Algorithm 3 and the

---

3We implement both algorithms with their chosen parameters to generate the adversarial examples, and compare the quality of sentences in the appendix. While in Section 5.2 we use the same word neighboring sets for all algorithms to make a fair comparison of only the optimization schemes.
Table 4. Human-subject validation. Task I measures classification accuracy while Task II the subjective likelihood that each example was crafted by a human (scale from 1 to 5). We used five participants, each shown \( n = 60 \) text examples, half original and half generated using our algorithm. The quality of the generated adversarial text (Task II) is near equal to the original and in fact, slightly higher for the Yelp dataset, but this finding is not necessarily statistically significant.

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Task I</th>
<th></th>
<th>Task II</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>News</td>
<td>Trec07p</td>
<td>Yelp</td>
<td>News</td>
</tr>
<tr>
<td>Original</td>
<td>70.0%</td>
<td>80.0%</td>
<td>100.0%</td>
<td>3.06 ( \pm ) 0.67</td>
</tr>
<tr>
<td>Adversarial</td>
<td>50.0%</td>
<td>80.0%</td>
<td>100.0%</td>
<td>3.13 ( \pm ) 0.50</td>
</tr>
</tbody>
</table>

Table 5. Performance of adversarial training.

<table>
<thead>
<tr>
<th>Dataset</th>
<th>LSTM</th>
<th>WCNN</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>News</td>
<td>Trec07p</td>
</tr>
<tr>
<td>Text (before)</td>
<td>93.3%</td>
<td>99.7%</td>
</tr>
<tr>
<td>Text (after)</td>
<td>94.5%</td>
<td>99.5%</td>
</tr>
<tr>
<td>ADV (before)</td>
<td>16.5%</td>
<td>31.1%</td>
</tr>
<tr>
<td>ADV (after)</td>
<td>32.7%</td>
<td>50.1%</td>
</tr>
</tbody>
</table>

Other two techniques: the gradient method (Gong et al., 2018) and the objective-guided greedy method (Kuleshov et al., 2018). To make a fair comparisons of the optimization schemes, we do not conduct sentence level paraphrasing in any of the methods, and we use the same hyperparameters and settings as suggested in Section 6.1. From Table 3 we can see that our method requires only 1/5 to 1/3 time cost relative to the objective-guided greedy method and also achieves better success rate. On the other hand, gradient method fails to produce good performance when we allow a small set of word replacements.

6.5 Human Evaluation Validation

Despite the significantly higher attack proportion of our text examples, our aim is to deliver a message that is faithful to and coherent with the original text. To evaluate the quality of these generated text examples, we presented a number of original and adversarial text pairs (randomly shuffled before the test) to five human evaluators. The evaluators were asked to complete two tasks: I) Assign the correct label to each text sample; II) Rate each text sample with respect the the likelihood that was crafted by a human (scale from 1 to 5). We adopted a majority vote for task I, and averaged the results from five evaluators for task II. As shown in Table 4, we found that human evaluators tend to achieve similar performance for each kind of text in both tasks, indicating that text examples generated via joint sentence and word paraphrasing are indeed coherent and faithful to the original texts in the relevant respects.

6.6 Adversarial Training.

Finally, we investigated whether our adversarial examples can help improve model robustness. For each dataset, we randomly selected 20% of the training data and generated adversarial examples from them using Algorithm 1. We then merged these adversarial examples with corrected labels into the training set and retrained the model. We present the testing and adversarial accuracy before and after this adversarial training process in Table 5. Under almost all circumstances, adversarial training improved the generalization of the model and made it less susceptible to attack.

7 CONCLUSION

In this paper, we propose a general framework for discrete attacks. Mathematically, we formulate the adversarial attack as an optimization task on a set of attacks. We then theoretically prove that greedy method guarantees a \( 1 - 1/e \) approximation factor for two classes of neural network for text classification task. Empirically, we propose a gradient-guided greedy method that inherits the efficiency of gradient method and ability to attack of greedy method. Specifically, we investigate joint sentence and word paraphrasing to generate attacking space that maintain the original semantics and syntax for text adversarial examples.

8 ACKNOWLEDGEMENTS

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REFERENCES


A PROOFS

A.1 Proof of Proposition 1

Proof of Proposition 1. We will show that even for a very simple function \( f \), it could be reduced to subset sum problem when \( k \geq 2 \).

For instance, let

\[
f(S) = \arg \max_{\supp(1) \subset S} \left\| \sum_{i=1}^{n} V(x_i^{(1)}) - v \right\|_2^2,
\]

where the target is to find the best \( \ell_2 \) approximation of some target vector \( v \) from the embedding vectors.

For simplicity, denote the embedding vector of each paraphrased words to be \( V(x_i^{(j)}) = v_i^{(j)}, 1 \leq i \leq n, 0 \leq j \leq k - 1 \). Suppose there is an algorithm that solves the above problem in time polynomial to \( n \). Then we will now show that the subset sum problem has a solution in polynomial time. Let the \( n \) numbers to be \( s_1, s_2, \ldots, s_n \), and the target to be \( W \). Then we let \( V(x_i) = [s_i, 0, 0, \ldots, 0] \), and \( V(x_j) = 0, j = 1, \ldots, k - 1 \), with target \( v = [W, 0, 0, \ldots, 0] \). Then just check if the best approximation of \( v \) is exactly \( v \) will suffice the subset sum problem. Therefore it contradicts with the fact that subset sum is in NP-complete class. \( \square \)

A.2 Proof of Proposition 2

Proof of Proposition 2. Define set function \( h(S) = \arg \max_{\supp(1) \subset S} V(T_1(x)) \top \nabla C_y(v) \), where \( v = V(x) \). Denote \( g = \nabla C_y(v) \). When \( V \) is bag-of-words embedding, we denote the embedding of each paraphrased word in \( x = [x_1, x_2, \cdots, x_n] \) as \( V(x_i^{(j)}) = e_d_{ij} \). Here for any \( i, e_i \) is defined as the one-hot vector with 1 in index \( i \) and 0 elsewhere. Then \( V(T_1(x)) = \sum_{i=1}^{n} e_d_{di_i} \).

\[
h(S) = \arg \max_{\supp(1) \subset S} V(T_1(x)) \top g = \arg \max_{\supp(1) \subset S} \sum_{i=1}^{n} e_d_{di_i} \top g = \arg \max_{\supp(1) \subset S} \sum_{i=1}^{n} g_{d_{di_i}} = 1 \top_S w,
\]

where \( w_i = \max_{0 \leq t \leq k-1} g_{d_{ti}} \).

When \( V \) is \( d \)-dimensional word2vec embedding, the embedding \( V(x) = [V(x_1) \top | V(x_2) \top | \cdots | V(x_n) \top] \top \in \mathbb{R}^{nd} \). Denote \( g_i = g_{(id-d+1):id} \) to be the gradient with respect to the word \( w_i \).

\[
h(S) = \arg \max_{\supp(1) \subset S} V(T_1(x)) \top g
= \arg \max_{\supp(1) \subset S} \sum_{i=1}^{n} V(x_i^{(1)}) \top g_i
= \arg \max_{\supp(1) \subset S} \sum_{i=1}^{n} V(x_i^{(1)}) \top g_i
= 1 \top_S w
\]

where \( w_i = \max_{0 \leq t \leq k-1} V(x_i^{(t)}) \top g_i \). Therefore for both bag-of-words embedding and word2vec embedding, \( h \) is a modular (linear) set function, and Problem 2 is solvable in polynomial time.

A.3 Proof of Claim 1

Proof of Claim 1. Clearly for any \( S \subset V \subset [n] \),

\[
f(S) = \max_{\supp(1) \subset S} C_y(V(T_1(x)))
\leq \max_{\supp(1) \subset V} C_y(V(T_1(x)))
= f(V)
\]

(since \( S \subset T \))

Therefore the set function \( f \) is non-decreasing. Since the problem of maximizing a monotone submodular function subject to a cardinality constraint admits a \( 1 - 1/e \) approximation algorithm(Nemhauser et al., 1978), Problem 1 can be solved in time polynomial to \( n \) with greedy method. \( \square \)

A.4 Proof of Theorem 1

Proof of Theorem 1. We start from a simple case, \( h = 1 \), i.e., a unit kernel size, and we look at a single feature corresponding to one filter, i.e. \( \hat{c}_j = \max_{i=1}^{n} c_{ij} \).

To further incorporate the transformation to the input, we rewrite \( \hat{c}_j \) as a function of the transformation index \( I \).

\[
\hat{c}_j(I) = \max_{i=1}^{n} \phi(w_j \top V(x_i^{(1)}) + b_j)
= \max_{i=1}^{n} v_{ij}^{(k)}
\]

where \( w_j \) is the \( j \)-th filter and we denote \( v_{ij}^{(k)} = \phi(w_j \top V(x_i^{(k)}) + b_j) \) for simplicity.

Let \( S, T \) denote two sets that satisfy \( S \subset T \subset [n] \). For any two vectors \( l^S \) and \( l^T \) satisfy that \( l^S_i = l^T_i, \forall i \in S \), and \( \supp(l^S) = S \), \( \supp(l^T) = T \). With the assumption that \( w_j \top V(x_i) \leq w_j \top V(x_i^{(t)}) \), and since the activation function is non-decreasing, we have \( v_{ij}^{(t)} \leq v_{ij}^{(k)}, \forall i \in [n], j \in [m], t \in [k-1] \), and thereby \( \hat{c}_j(I^S) \leq \hat{c}_j(I^T) \).

Therefore for any new element’s position \( s \) and its replace-
ment index $t$, we have

$$
\hat{c}_j(t^S + te_s) - \hat{c}_j(t^S) = \max \{v^{(t)}_{x_j} - \hat{c}_j(t^S), 0\} \\
\geq \max \{v^{(t)}_{x_j} - \hat{c}_j(T^T), 0\} \\
\text{(since } \hat{c}_j(t^S) \leq \hat{c}_j(T^T)) \\
= \hat{c}_j(T^T + te_s) - \hat{c}_j(T^T).
$$

Since the final output probability is a positive weighted summation of each $\hat{c}_j$, it also satisfies

$$
C_{\text{WCNN}}(t^S + te_s) - C_{\text{WCNN}}(t^S) \\
\geq C_{\text{WCNN}}(T^T + te_s) - C_{\text{WCNN}}(T^T)  	ag{6}
$$

Taking the max over all $t^S, T^T$ we have:

$$
f(S + \{s\}) = \max_{t=1}^{T} \max_{\text{supp}(t) = S} C_{\text{WCNN}}(t^S + te_s)
$$

Therefore

$$
f(S + \{s\}) - f(S) = \max_{t=1}^{T} \frac{C_{\text{WCNN}}(t^S + te_s) - C_{\text{WCNN}}(t^S)}{\text{supp}(t) = S}
$$

$$
\geq \max_{t=1}^{T} \frac{C_{\text{WCNN}}(T^T + te_s) - C_{\text{WCNN}}(T^T)}{\text{supp}(t) = T}
$$

$$(\text{from (6)})
$$

$$
f(T + \{s\}) - f(T).
$$

The case when $2 \leq h \leq s$ is essentially the same with $h = 1$ since each window has no overlapping. We could simply replace $\nu_1$ by $\nu_{1:h}$ and conduct the same analysis.

\[\square\]

### A.5 Proof of Theorem 2

**Proof of Theorem 2.** Recall that the hidden state node $h_i$ is defined recursively as:

$$
h_0 = C,  \quad (C \text{ is constant})
$$

$$
h_i = \phi(w_h x_{i-1} + m^T V(x_{i-1}) + b).
$$

And the classifier output is $C_{\text{RNN}}(V(x)) = y h_T$.

For simplicity, we denote $v^{(j)}_{x_i} = m^T V(x^{(j)}_{x_i}) + b$. Since we will only look for the classifier output, without loss of generality, we assume $v^{(0)}_{x_i} \geq v^{(0)}_{x_i}, \forall i \in [T], j \in [k-1]$.

For a fixed input $x = [x_1, x_2, \ldots, x_T]$ and transformation index $l$, we want to study how changing an intermediate hidden state affects the consecutive layers’ output. Therefore we represent the value of a $j$-th hidden state as a function of the $i$-th hidden node and the transformation label $l$, that captures the network from $i$-th through $j$-th time steps, i.e.,

$$
f_{i:j}(h_i, l) = \phi(w \cdot \phi(w_h x_i + v^{(l)}_{x_i}) + \cdots + v^{(l)j}_{x_i-j+1}).
$$

Finally we want to study the whole network’s output $y_{f:0:T}(C, l)$. We first prove the following lemma:

**Lemma 1.**

$$
f_{i:j}(h_i + \delta, l) - f_{i:j}(h_i, l) \\
\geq f_{i:j}(h_i + \delta, l + te_s) - f_{i:j}(h_i + l + te_s).  \tag{7}
$$

for any $0 \leq i < j \leq T, t \in [k-1], s \in [T], s \notin \text{supp}(l), \delta > 0$.

**Proof of Lemma 1.**

$$
f_{i:j}(h_i + \delta, l + te_s) \\
= f_{s+1:j}(\phi(w_{f:s}(h_i + \delta, l + te_s) + v^{(s)}_{x}), l + te_s)
$$

Now we simplify the equation by define $a(\delta, t) = \phi(w_{f:s}(h_i + \delta, l + te_s) + v^{(s)}_{x}), l + te_s)$

$$
a(\delta, t) - a(0, t) \leq a(\delta, 0) - a(0, 0).  \tag{8}
$$

Now since $f_{s+1:j}(., l)$ is a composite of concave function and is also concave, we have:

$$
f_{i:j}(h_i + \delta, l + te_s) - f_{i:s}(h_i, l + te_s) \\
= f_{s+1:j}(a(\delta, l), 1) - f_{s+1:j}(a(0, l), 1)
$$

$$
\leq f_{s+1:j}(a(\delta, 0) + a(0, 0), l) - f_{s+1:j}(a(0, 0), 1)
$$

$$
\leq f_{s+1:j}(a(\delta, 0), 1) - f_{s+1:j}(a(0, 0), 1)
$$

$$
\text{(from (8) and non-decreasing $f_{s+1:j}(., 1)$)}
$$

$$
= f_{s+1:j}(a(\delta, 0), 1) - f_{s+1:j}(a(0, 0), 1)
$$

$$
\text{(from concavity of $f_{s+1:j}(., 1)$)}
$$

$$
f_{i:j}(h_i + \delta, l) - f_{i:s}(h_i, l)
$$

\[\square\]

Lemma 1 could be extended to a more general form. Suppose two indices $l^S$ and $l^U$ satisfy $\text{supp}(l^S) = S, \text{supp}(l^U) = U, S \subset U$. Let $l_i^{1:S} = l^{1:S} = l_i, \forall i \in S$. Since we could write $l^U$ as $l^U = \sum_{i \in U \setminus S} l_i^{1:S} e_i$, by repeatedly using Lemma 1 we have:

$$
f_{i:j}(h_i + \delta, l^S) - f_{i:j}(h_i, l^S) \\
\geq f_{i:j}(h_i + \delta, l^U) - f_{i:j}(h_i, l^U),  \tag{10}
$$

This conclusion basically claims an increase into an intermediate layer of the network will have smaller effect to the
output when the network is attacked on more time steps. Then back to Theorem 2. Now consider we add a coordinate s to the set S and U, s \notin \text{supp}(S) \cup \text{supp}(U).

\[ f_{0:T}(C, 1^S + te_s) - f_{0:T}(C, 1^S) \]

\[ = f_{0:T}(\phi(wf_{0:S-1}(C, 1^S) + v_s^{(o)}), 1^S) \]

\[ - f_{0:T}(\phi(wf_{0:S-1}(C, 1^S) + v_s^{(0)}), 1^S) \]

\[ \geq f_{0:T}(\phi(wf_{0:S-1}(C, 1^S) + v_s^{(0)}), 1^U) \]

\[ - f_{0:T}(\phi(wf_{0:S-1}(C, 1^S) + v_s^{(0)}), 1^U) \]

(from (10) and since \( \phi \) is non-decreasing, \( v_s^{(t)} \geq v_s^{(0)} \))

\[ \geq f_{0:T}(\phi(wf_{0:S-1}(C, 1^U) + v_s^{(t)}), 1^U) \]

\[ - f_{0:T}(\phi(wf_{0:S-1}(C, 1^U) + v_s^{(0)}), 1^U) \]

(since \( f \) is concave and similar analysis as (9))

\[ = f_{0:T}(C, 1^U + te_s) - f_{0:T}(C, 1^U) \]  

(11)

Finally, since

\[ \max_{\text{supp}(1) \subset S \cup \{s\}} C^{RNN}(V(T_1(x))) \]

\[ = \max_{\text{supp}(1) \subset S \cup \{s\}} \max_{t \in [k-1]} yf_{0:T}(C, 1^S + te_s), \]

we have:

\[ \max_{\text{supp}(1) \subset S \cup \{s\}} C^{RNN}(V(T_1(x))) \]

\[ - \max_{\text{supp}(1) \subset S} C^{RNN}(V(T_1(x))) \]

\[ = \max_{\text{supp}(1) \subset S \cup \{s\}} \left( \max_{t \in [k-1]} yf_{0:T}(C, 1^S + te_s) - yf_{0:T}(C, 1^S) \right) \]

\[ \geq \max_{\text{supp}(1) \subset U \cup \{s\}} \left( \max_{t \in [k-1]} yf_{0:T}(C, 1^U + te_s) - yf_{0:T}(C, 1^U) \right) \]

(since (11) holds for any t)

\[ = \max_{\text{supp}(1) \subset U \cup \{s\}} C^{RNN}(V(T_1(x))) \]

\[ - \max_{\text{supp}(1) \subset U} C^{RNN}(V(T_1(x))) \]

\[ \square \]

B Data statistics

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Task</th>
<th>#Train</th>
<th>#Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trec07p</td>
<td>Spam filtering</td>
<td>67.9k</td>
<td>7.5k</td>
</tr>
<tr>
<td>Yelp</td>
<td>Sentiment analysis</td>
<td>560k</td>
<td>38k</td>
</tr>
<tr>
<td>News</td>
<td>Fake news detection</td>
<td>5.3k</td>
<td>1.0k</td>
</tr>
</tbody>
</table>

Table 6. Statistics of each datasets

Perplexity Evaluation We measure and compare perplexity (Brown et al., 1992) of the adversarial and the original testing sets to verify the quality and fluency of our examples. The perplexity measured relies on the language model \( P \) trained from each training set. We represent the perplexity as:

\[ \frac{1}{|X|} \sum_{x \in X} \ln(P(x)) \]

where \( x \) is all the 3-grams in the document dataset. Normally the evaluation should be on the document, but our method naively show better (smaller) perplexity than original dataset since sentence paraphrasing usually shrinks the sentence length. Therefore in order to make fair comparisons, we computed the average probability of 3-grams instead of the whole document. Results are shown in Table 7 and prove that our method preserves or even improves the coherence in the adversarial examples compared with the clean datasets.

<table>
<thead>
<tr>
<th>Dataset</th>
<th>News</th>
<th>Trec07p</th>
<th>Yelp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Origin</td>
<td>11.1</td>
<td>5.1</td>
<td>11.0</td>
</tr>
<tr>
<td>ADV(LSTM)</td>
<td>10.9</td>
<td>5.3</td>
<td>10.3</td>
</tr>
<tr>
<td>ADV(WCNN)</td>
<td>11.1</td>
<td>4.8</td>
<td>11.2</td>
</tr>
</tbody>
</table>

Table 7. Perplexity comparisons of the original testing set and adversarial examples for both LSTM and WCNN classifier.

C Comparisons with other methods with concrete examples

In this section, we provide some concrete examples to compare our method with the other related methods. The following six examples respectively show the combinations of three datasets (fake news, Trec07p, and Yelp) as well as the two models we use (LSTM and WCNN).

We use red font to denote changes from sentence level paraphrasing and blue for word paraphrasing.

C.1 Empirical example 1: Task - Fake news detection.
Classifier-CNN.

Method: Ours. Origin: 100% Real. ADV: 71% Fake

6 Six detainees detained in raids in Belgium Brussels, Belgium (CNN) Police detained six people in raids Thursday night as when investigators raced were sent to uncover the network behind this week’s terror attacks in the Belgian capital. The Belgian federal prosecutor’s office didn’t provide details about who had been detained in the Brussels raids, why they had been apprehended or whether they will face charges. It will be decided tomorrow if these people will remain in custody, the office said in a statement released late Thursday. Two people were taken into custody in Brussels’ Jette neighborhood, one person was detained in a different part of the capital, and three people were in a vehicle in front of the federal prosecutor’s office when authorities apprehended them, public broadcaster RTBF reported. So far, authorities have Authorities said they believe five men played a part people took a shot in Tuesday’s bombings in Belgium that killed 31 people and injured wounded 330. Three of the attackers are dead. Two of them could still be
on the loose. Investigators are combing over evidence from surveillance footage and the explosives stash they seized from an apparent hideaway in a suburb. Sweeps where investigators detain people first and ask questions later are likely to become an increasingly common tactic, CNN national security analyst Juliette Kayyem said. There will be lots more of them, she said. They are going to be what’s called overbroad. They are going to just try to find people or evidence that may stop the next terrorism attack, and they will figure out who they have under custody. Khalid El Bakraoui, one of the terrorists who bombed a train near the Maelbeek metro station, is dead. Authorities believe a second unidentified person was also involved in that attack, a senior Belgian security source told CNN. But investigators don’t know where the suspect is – or whether he’s dead or alive. Surveillance footage shows the man holding a large bag at the station, according to Belgian public broadcaster RTBF. It’s not clear if he was among the at least 20 killed in that blast, RTBF said. Authorities have released a grainy image of another suspect who they believe is on the run. That man, they say, shown in photographs wearing a black hat, was one of three attackers at Brussels Airport. Authorities say he planted a bomb at the airport and left. The other two men in the photographs are believed to be the suicide bombers. Fair to ask whether ‘we missed the chance’ Did Belgian authorities miss a chance to stop at least one of the suspects involved in the attacks? Bakraoui had been sentenced to nine years in prison in Belgium back in 2010 for opening fire on police officers with a Kalashnikov during a robbery, according to broadcaster RTBF and CNN affiliate RTL. Needless to say, he didn’t serve all that time. Given the facts, it is justified that ... people ask how it is possible that someone was released early and we missed the chance when he was in Turkey to detain him, said Jambon, whose offer to resign was rebuffed by Prime Minister Charles Michel. Investigators suspect Abdeslam planned to be part of an attack by the same ISIS cell that lashed out Tuesday, a senior Belgian counterterrorism official told CNN’s Paul Cruickshank. Authorities looked Wednesday at the Brussels homes of the Bakraoui brothers. **These two searches findings were not conclusive decisive, the federal prosecutor’s office prosecutors said.** Homes were searched Thursday in several areas in and around the city, officials said. One operation in the neighborhood of Schaerbeek stretched for hours into Friday morning. Investigators sealed off streets for several blocks. It was not immediately clear why such a large area had been cordoned. Masked teams in hazmat gear could be seen exiting a building and heading toward a police van. As investigations continue, a larger question looms: What could happen next? Not long ago, Western authorities believed ISIS was focused on taking territory in Syria and Iraq, not lashing out elsewhere. But U.S. officials now think the extremist group has been sending trained militants to Europe for some time. These men don’t necessarily follow orders directly from ISIS headquarters. But they build on what they’ve learned, as well as a shared philosophy and approach, to develop their own terror cells and hatch their own plots. How many more ISIS militants are in Europe, poised to attack? That’s not clear. For now, though, the top priority is tracking down the two men linked directly to Tuesday’s terror.

**Method:** Greedy(Kuleshov et al., 2018). **Origin:** 100% Real. **ADV:** 79% Fake

6 7 detained detention in raids in Belgium Brussels, Belgium (CNN) **Police cops detained deported** six people in raids Thursday night as investigators investigation raced to uncover the network behind this week’s terror terrorists attacks in the Belgian capital. The Belgian federal prosecutor’s office didn’t provide details about who had been detained in the Brussels raids, why they had been apprehended or whether they **we will should face** charges. It will be decided tomorrow if these people will remain in custody, the office said said in a statement stating released late Thursday. Two people were taken into custody in Brussels’ Jette neighborhood, one person was detained detention in a different part of the capital, and three people were in a vehicle in front of the federal prosecutor’s office when authorities apprehended them, public broadcaster RTBF reported. So far, authorities have said they believe five men played a part in Tuesday’s bombings in Belgium what killed wounded 44 26 people individuals and injured 330. Three of the attackers are dead. Two of them could still be on the loose. Investigators Investigating are they combing over evidence from surveillance footage filmed and the explosives stash they never never seized from an apparent obvious hideaway in a suburb. Sweeps where investigators detain people first and ask questions later are likely to become an increasingly common commonly tactic, CNN national Security analyst analysts Juliette Kayyem said said. There will be lots more of them, she knew said say. They are going to be what’s called overbroad. They are going to just try to find people or or evidence findings that may stop the the of of next before Terrorism attack, and they will figure out who they have under custody. Khalid El Bakraoui, one of the terrorists who bombed a train near the Maelbeek metro station, is dead. Authorities believe a second unidentified person was also involved in that attack, a senior Belgian Security source sources told saying CNN. But investigation don’t know where that suspect victim is be – or any whether not he’s dead dying or and/or alive. Surveillance footage shows the man holding a large bag at the station, according to Belgian public broadcaster RTBF. It’s not clear if he was among the at least 20 26 killed kill in that blast, RTBF said say. Authorities have released a another grainy image of the another suspect who they believe is on the run. That man, they say, shown in photographs photo wearing wear a one black red hat, was one of three twelve attackers at-
6 detained in raids in Belgium Brussels, Belgium (CNN) Police detained six people in raids Thursday night as well investigators raced to uncover the network behind this week’s terror attacks in the capital. The Belgian federal prosecutor’s office didn’t provide details about who had been detained or whether they will be face charges. It should decided tomorrow if these people will remain in custody, the offices said in a statement released late Thursday. The three people were taken into custody in Brussels’ Jette neighborhood, another person was detained in a different part of the capital, and three people were had in a vehicle in front of the federal prosecutor’s office when investigators apprehended them, public broadcaster RTBF reported. So far, authorities have said they believe five men played a part in Tuesday’s bombings in Belgium that killed 31, 29 people and injured 330. Three of the attacker be dead. Two of them could still be on the loose. Investigators are combing over evidence from surveillance footage and the explosives stash they seized from an apparent hideaway in a suburb. "Sweeps where investigators detain people and ask questions later are likely to become an increasingly common tactic," CNN national security analyst Juliette Kayyem said. There will be lot more of them, she said. They are going to be what’s called known overbroad. They are going able just trying to find people or evidence that may stop the next attack, and they will figures out up whom they have under custody. Khalid El Bakraoui, one of the terrorists who bombed a train near the Maelbeek metro station, is dead. Authorities believe a second unidentified person was also involved in that attack, a senior Belgian Security sources told talk CNN. But though investigators don’t know where there that because suspect is – or whether he’s dead dying or either alive. Surveillance footage shows the man holding a large bag at the station, according to Belgian public broadcaster RTBF. It’s not clear if he was among the of at least 20 25 killed in that because blast, RTBF said guess. Authorities have’ve released another image of another one suspect who they have believe is on the run. That man, they say, shown in photographs wearing a black grey hat, was one of three attackers at Brussels Airport. Authorities say he planted a bomb at the airport and left. The other two women in of photographs are believed to be the suicidal bombers. Fair to ask tell whether ‘we missed the chance’ Did Belgian authorities miss a chance to stop at least one of the suspects involved in the attacks? Bakraoui had been sentenced to nine years in prison in Belgium back in 2010 for opening fire on policemen officers deputies with a Kalashnikov during a robbery, according to broadcaster RTBF and CNN affiliate RTL. Needless to say, he didn’t serve all that time. Given the facts, it is justified that ... people everyone ask tell how it is possible that what someone was released early and we just missed of chance when he was in Turkey to detain him, said told Jambon, whose offer to resign was rebuffed by Prime Minister Charles Michel. Investigators investigation suspected Abdeslam planned planning to be part of an attack enemy by the same different ISIS cells that lashed out Tuesday, a senior Belgian counter-terrorism unofficial told CNN’s Paul Cruickshank. Authorities looked Wednesday at the Brussels homes of the Bakraoui brothers. Those Them these one searches were not conclusive, the federal prosecutor’s Office said. One operation in the neighborhood of Schaarbeek stretched for hours for Friday morning. Investigators sealed off streets for several blocks. It was not immediately clear why such a large area had been cordoned. Masked teams in hazmat gear could be seen exiting a building and heading toward a police van. As investigations continue, a larger question looms: What could happen next? Not long ago, Western authorities believed ISIS was focused on taking territory in Syria and Iraq, not lashing out elsewhere. But U.S. officials now think the extremist group has been sending trained militants to Europe for some time. These men don’t necessarily follow orders directly from ISIS headquarters. But they build on what they’ve learned, as well as a shared philosophy and approach, to develop their own terror cells and hatch their own plots. How many more ISIS militants are in Europe, poised to attack? That’s not clear. For now, though, the top priority is tracking down the two men linked directly to Tuesday’s terror.

Method: Gradient method(Gong et al., 2018). Origin: 100% Real. ADV: 99.5% Real
ties looked seemed Wednesday at the Brussels homes of the Bakraoui brothers. Those two searches were not conclusive, the federal prosecutor’s office said. Homes were searched Thursday in several numerous areas in and around the city, officials said guess. One operation in the of neighborhood of Schaerbeek stretched for hours into Friday morning. Investigators sealed off streets for several blocks. It was not immediately clear why such a large areas had been being cordoned. Masked teams in hazmat gears could be seen exiting another buildings and heading towards another police van. As investigations continue, a larger sized questions looms: What could happen next? Not long ago, Western authorities believed ISIS was focused on taking territory in Syria and Iraq, not lashing out elsewhere. But U.S. officials now think know the extremist group has been sending trained militants insurgents to Europe for some time. These men don’t necessarily follow orders directly from ISIS headquarters. But they build on what they’ve learned, as well as a shared philosophy and approach, to develop their own terror cells and hatch their own your plots. How many more less ISIS militants are these in Europe, poised to attack? That’s not clear. For now, though, the top priority is tracking down the two men linked directly to Tuesday’s terror.

C.2 Empirical example 2: Task - Fake news detection. Classifier - LSTM.

Method: Ours. Origin: 100% Fake. ADV: 77% Real
Man punctuates high-speed chase with stop at In-N-Out Burger drive-thru Print [Ed. - Well, that’s a new one. Okay, that’s a new one.] A One man is in custody after leading police on a bizarre chase into the east Valley on Wednesday night. Phoenix police began following the suspect in Phoenix and the pursuit continued into the of east Valley, but it took a bizarre turn then when that the of suspect stopped at In-N-Out Burger restaurant’s drive-thru nearby Priest and Ray Roads in Chandler. The suspect appeared to ordering food, but then drove away and got out of his pickup truck near Rocks Wren Chickadee Ways and Ray Road. He then ran into a backyard and tried to get into a house through the back again door.

C.3 Empirical example 3: Task - Spam filtering. Classifier - WCNN.

Method: Ours. Origin: 100% Spam. ADV: 77% Ham
Become Fit For Life! HGH is a very complex molecule produced by the anterior lobe of the pituitary gland, which is located at the base of the brain. While it stimulates growth in children, it is important for maintaining a healthy body composition and well-being in adults. It is the primary hormone estrogen that controls many several of the body’s organs and it stimulates tissue repair, brains functions, cell replacement, and enzyme function. Determining the levels of IGF-1 (Insulin Growth Factor) is how we measure HGH in the body. Receive a younger future potential with HGH

Method: Greedy(Kuleshov et al., 2018). Origin: 100% Spam. ADV: 71% Ham
Become Fit For Life! HGH is a very fairly complex molecule produced by the anterior lobe of the pituitary gland, which is has located at the base of the brain. While it stimulates growth growing in children, it what is important significant for maintaining another healthy body composition and well-being in adults. It is the primary secondary hormone progesterone that could controls many several of the body’s organs and it that stimulates tissue repair, brains functions, cell replacement, and enzyme function. Determining Determine the levels of IGF-1 (Insulin Growth Factor) is which how understand we measure HGH in the of body. Receive a younger future with HGH

Method: Gradient method(Gong et al., 2018). Origin: 100% Spam. ADV: 1 – $2.7e^{-5}$ spam
Become Fit For Life! HGH is a very complex molecule produced by the anterior lobe of the pituitary gland, which
that is located situated at the base of the brain. While it stimulates growth in children, it but is has important for maintaining a healthy body compositions and well-being in adults. It is the primary secondary hormones that controls many of the body’s organs and it but stimulates tissues repair, brains functions, cell replacement, and enzyme function. Determining the levels of IGF-1 (Insulin Growth Factor) is how we measure HGH in the of body. Receive a younger future with HGH


Classifier - LSTM.

Method: Ours. Origin: 100% Ham. ADV: 87% Spam
I’ve always run jigdo-lite against my own mirror. It provides offers two couple things:
1) Proves you can reliably build the ISOs from what I have mirrored locally.
2) Doesn’t waste additional bandwidth. As long as the checksums match what is provided from the official ISO image masters site, I don’t see what the difference would be. Anyone else do this? ↓ ^_^
Will Simon Paillard wrote:
> On Mon, Apr 09, 2007 at 08:43:07AM -0400, Jean-Francois Chevrette wrote:
>> Hi,
>>>
>>> does anyone have another straightforward guide on how to use jigdo to build and mirror ISOs? I’ve been reading both jigdo documentation and debian’s homepage web-site on the subject and it just won’t work.
>>>
>>> May with this one:
http://www.debian.org/CD/mirroring/
jigdomirror
>>> and the related links?
>>
>>>
Best regards,

Method: Greedy(Kuleshov et al., 2018). Origin: 100% Ham. ADV: 90% Spam
I’ve always run jigdo-lite against my own mirror. It provides offers two five things:
1) Proves you can reliably build the ISOs from what I have mirrored locally.
2) Doesn’t waste additional extra bandwidth. As long as the checksums match what is provided from the unofficial ISO image master site, I don’t see what the difference would be. Anyone else do this? ↓ :-)
Will Simon Paillard wrote:
> On Mon, Apr 09, 2007 at 08:43:07AM -0400, Jean-Francois Chevrette wrote:
>> Hi,
>>>>
>>> does anyone have another straightforward guide on how to use jigdo to build and mirror ISOs? I’ve been reading both jigdo documentation and debian’s homepage web-site on the subject and it just won’t work.
>>> May with this one:
http://www.debian.org/CD/mirroring/
jigdomirror
>>> and the related links?
>>>>
Best regards,

C.5 Empirical Example 5: Task - Sentiment analysis.

Classifier - CNN.

Method: Ours. Origin: 100% Positive. ADV: 93% Negative
This Starbucks location is located in the Bally’s Grand Bazaar Shops. It’s open 24/7 and it is huge. There is plenty of seating. Most of the seating is stadium type seating with benches. They also have an outdoor patio. The staff is very friendly and attentive to the guests. I do notice that they are understaffed sometimes when they are busy. They’ll get your drinks out pretty fast though. Also, this location is not owned by the casino property so they don’t charge outrageous prices like the location as a place on the Linq promenade does. Definitely one of my favorite Starbucks stores. Stop by if you’re on the Strip.
Method: Greedy(Kuleshov et al., 2018). Origin: 100% Positive. ADV: 74% Negative
This Starbucks location is be located in the Bally’s Grand Bazaar Shops. It’s open 24/7 and is Nothing is be huge. There Nothing is plenty of the seating. Most Extremely of the seating is has stadium types seating seats with benches. They Have also will have never an out door patio. The staff is very friendly and attentive to the guests. I do notice that they are under staffed sometimes when they are busy. They get your drinks out pretty fast though. Also, this location is not owned by the casino so they don’t charge outrageous prices like the location on the Linq promenade does. Definitely one of my favorite Starbucks stores. Stop by if your on the Strip.

Method: Gradient method(Gong et al., 2018). Origin: 100% Positive. ADV: 1 – 6.9e−12 Positive This It Starbucks McDonalds location is located in the Bally’s Grand Bazaar Shops. It’s open 24/7 and it is huge. There is plenty of seating. Most Many of the seating is stadium type seating with benches. They also have ‘ve an out up door patio. The staff is very friendly and attentive to the guests. I do notice that they are under staffed sometimes when they are busy. They getting your drinks out pretty fast though. Also, this location is not owned by the of casino so too they don’t charge outrageous prices like think the location on the of Linq promenade seafront does. Definitely one of my favorite Starbucks stores. Stop by if unless your on the Strip.

C.6 Empirical Example 6: Task - Sentiment analysis. Classifier - LSTM.

Method: Ours. Origin: 100% Positive. ADV: 93% Negative
I suppose I should write a review here since my little Noodle-oo is currently serving as their spokes dog in the photos. We both love Scooby Do’s. They treat cure my little butt-faced dog like a prince and are receptive to correcting anything about the cut that I perceive as being weird. Like that funny humorous poofy pompadour. Mohawk it out, yo. Done. In like five seconds my little woman was looking fabulous and bad ass. Not something easily accomplished with a prancing pup that literally chases butterflies through tall grasses. (He ended up looking like a little lamb as the cut grew out too. So adorable.) The shampoo they use here is also amazing. Noodles usually smells like tacos (a combination of between beef stank and corn chips) but after getting back from the Do’s, he smelled like Christmas morning! Sugar and spice and everything nice instead of frogs and snails and puppy dog tails. He’s got some gender identity issues to deal with. The pricing is also cheaper than some of the big name conglomerates out there. I’m talking to you Petsmart! I’ve taken my other pup to Smelly Dog before, but unless I need dog sitting play time after the cut, I’ll go with Scooby’s. They genuinely seem to like my little Noodle monster.

Method: Gradient method(Gong et al., 2018). Origin: 100% Positive. ADV: 93% Negative
I suppose I should write Write a review here since my little Noodle-oo is currently serving as their spokes dog in the photos. We both love Scooby Do’s. They treat cure my little butt-faced dog like another prince knight and but are receptive to correcting anything about the cut that I perceive as that being weird. Like that funny poofy pompadour. Mohawk it out, yo. Done. In like five seconds my little man was looking fabulous and bad ass. Not something easily readlly accomplished with a prancing strutting pup that literally chases butterflies through tall grasses. (He ended up looking like a little lamb beef as the The cut grew out too. So adorable.) The shampoo they use here is also amazing. Noodles usually smells like tacos quesadillas (a combination of beef stank and corn chips) but after getting back from the Do’s, he smelled like Christmas morning! Sugar and spice cumin and everything nice instead of frogs and snails and puppy dog tails. He’s got some those gender sexuality identity issues difficulties to deal contract with. The pricing is also cheaper than some of the big huge name conglomerates out there. I’m talking to you Petsmart! I’ve taken brought my other pup to Smelly Dog before, but unless I need dog sitting play time after the cut, I’ll go with Scooby’s. They genuinely nonetheless seem to like my little Noodle monster.